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ASYMPTOTICALLY ROBUST DETECTION OF STOCHASTIC SIGNALS  
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STATION DEPT OF ELECTRICAL ENGINEERING  
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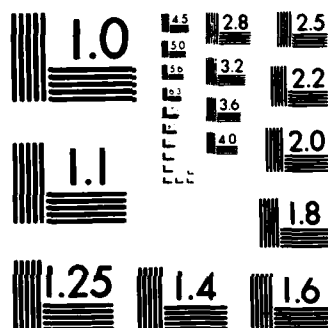
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ASYMPTOTICALLY ROBUST DETECTION OF STOCHASTIC  
SIGNALS IN CONTAMINATED NOISE

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ABSTRACT

We consider the discrete time detection of stochastic signals in white noise, where the univariate noise density is known perfectly only on an interval about the origin. We present a method to enhance the asymptotic performance of the detector by exploiting this knowledge, and at the same time preserve robustness properties of the detector to the remaining inexact knowledge of the univariate noise density via a saddlepoint condition. We then provide examples to show that improved performance is indeed obtained.

1. INTRODUCTION

Consider the discrete time detection of stochastic signals in independent and identically distributed noise. If the underlying statistical distributions are completely known, then selection of a Neyman-Pearson optimal detector is possible. However, in many cases knowledge of the univariate noise density is imperfect, particularly for extreme values of its argument. In such situations the employment of a detector which is robust to the inexact statistical knowledge may be desirable. Earlier work of Kassam and Thomas [1] and Poor, Mami, and Thomas [2] points toward the introduction of limiting for observations of large magnitude in order to impart robustness into the detection scheme. In particular, [2] considers the

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asymptotically robust detection of stochastic signals in imperfectly known noise.

In this paper we consider further applications of detectors possessing the same property of limiting. We note that often more is known about the univariate noise density near the origin than on the tails, and such knowledge has the potential to be exploited for improved performance. In some earlier work [3] we show how to design the robust detector by exploiting this knowledge for improved performance over the detector of [1]; this work is applicable to the case of a time varying deterministic signal embedded in imperfectly known noise. We now consider the analogous extension of [3] for improved performance over the detector of [2]. Our results illustrate how such improved performance can be obtained and still preserve the robustness of the detector to the remaining inexact knowledge of the univariate noise density through the employment of limiting.

## II. DEVELOPMENT

Suppose we denote the i.i.d. discrete time noise process by  $\{N_i; i=1, 2, \dots\}$ , with  $f(\cdot)$  denoting the associated univariate noise density, and let the stochastic signal be denoted by  $\{S_i; i=1, 2, \dots\}$ ; the zero mean independent signal process is assumed to be independent of the noise. The detection problem then reduces to a choice between

$$H_0 : Y_i = N_i ; i = 1, 2, \dots, n$$

$$H_1 : Y_i = N_i + \theta S_i ; i = 1, 2, \dots, n$$

where we observe realizations  $y_i$  of the random variables  $Y_i$ . As in [1-3], we assume the noise density  $f(\cdot)$  is a member of the Huber-Tukey mixture class, i.e.  $f(\cdot)$  can be expressed as  $f(\cdot) = (1-\epsilon)f_0(\cdot) + \epsilon h(\cdot)$ , where  $\epsilon$  is a fixed known number between zero and unity,  $f_0(\cdot)$  is a known symmetric nominal density, and  $h(\cdot)$  is an unknown "contaminating" density. In addition, however, we assume that  $f(x)$  is known exactly for each  $x$  on an

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interval  $[-a, a]$ , i.e.  $h(x) = f_0(x)$  for all  $x$  in  $[-a, a]$ . We note in passing that a similar assumption is employed in [2] without being directly exploited. Motivated by the results of [1-3], as well as the form of the locally optimal detector given by [2], we employ the class of detectors where a test statistic of form

$$T_g(y) = \sum_{i=1}^n E\{S_i^2\} g(y_i) \text{ is compared to a threshold, with the}$$

nonlinearity

$$g(\cdot) \text{ given by } g(x) = \begin{cases} f_0''(x)/f_0(x) & \text{if } |x| \leq k \\ f_0''(k)/f_0(k) & \text{otherwise} \end{cases}$$

We thus see that specifying  $k$  effectively specifies the detector.

In order to analyze and compare the performance of the detector, and hence determine  $k$  for best performance, an asymptotic fidelity criterion is employed. Because of its relationship to the Pitman-Noether theorem [4], the criterion of Asymptotic Relative Efficiency (ARE) is particularly useful; the associated efficacy functional  $\eta$  provides a convenient measure of performance, where  $\eta$  is defined by

$$\eta(k, h) = \frac{\left[ \int_{-\infty}^{\infty} g''(x) f(x) dx \right]^2}{\int_{-\infty}^{\infty} g^2(x) f(x) dx}, \text{ with}$$

$$c = \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n \left[ E \left\{ S_i^2 \right\} \right]^2}{n} \quad . \quad \text{As}$$

in other work which employs the ARE, the results of this paper are valid only when various mild regularity conditions are imposed on the underlying statistical distributions. These assumptions are imposed to guarantee that our expressions exist and, when necessary, are nonzero; additionally,

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regularity assumptions are required to insure the relevancy of the efficacy functional to the ARE via satisfaction of the hypothesis of the Pitman-Noether theorem [4]. In this paper, our results are presented under the tacit assumption of these numerous though obvious mild conditions. We refer the interested reader to [5-7], in which analogous assumptions are explicitly listed. It should be noted that we do not impose symmetry assumptions on the contaminating density  $h(\cdot)$ .

To preserve the robustness property of the detector,  $k$  is then chosen according to  $\sup_k \inf_h \eta(k, h)$ , where dependence of  $\eta$  on  $k$  and  $h(\cdot)$  is indicated. Such a choice of  $k$  will be called an optimal choice. The following result provides insight into how to obtain an optimal choice of  $k$ :

Theorem: Let

$K = \{k > 0: \text{ if } k > a \text{ then } |g(\cdot)| \text{ and } |g'(\cdot)| \text{ are weakly monotone on } [a, k]\}$ . Then a necessary and sufficient condition for  $k$  to be an optimal choice over  $K$  is that  $k$  yield the supremum among

$$(a) \quad \inf_h \eta(k, h) = \frac{2c \left[ \int_0^a g''(x) f_0(x) dx + (1-c) \int_a^k g''(x) f_0(x) dx \right]^2}{\left[ \int_0^a (g^2(x) - g^2(k)) f_0(x) dx + (1-c) \int_a^k (g^2(x) - g^2(k)) f_0(x) dx + \frac{1}{2} g^2(k) \right]}$$

$$(b) \quad \inf_h \eta(k, h) = \frac{2c \left[ \int_0^k g''(x) f_0(x) dx \right]^2}{\int_0^k (g^2(x) - g^2(k)) f_0(x) dx + \frac{1}{2} g^2(k)}$$

if  $k > a$

if  $k \leq a$ .

Proof: Consider first the proof for part (a). Note that by negative scaling when necessary we can assume without loss of

generality that  $g'(\cdot)$  is weakly increasing on  $[a, k]$ , i.e.  $g''(x) \geq 0$  on  $[a, k]$ . Moreover, by symmetry we also have  $g''(x) \geq 0$  on  $[-k, -a]$ . In a manner similar to the proof of the Theorem of [3] we obtain by direct evaluation

$$\eta(k, h) =$$

$$c \left[ 2 \int_0^a g''(x) f_0(x) dx + (1-\varepsilon) \int_a^k g''(x) f_0(x) dx + \varepsilon \left[ \int_a^k g''(x) h(x) dx + \int_{-k}^{-a} g''(x) h(x) dx \right] \right]^2$$

$$\frac{\left[ 2 \int_0^a (g^2(x) - g^2(k)) f_0(x) dx + 2(1-\varepsilon) \int_a^k (g^2(x) - g^2(k)) f_0(x) dx + g^2(k) + \varepsilon \left[ \int_a^k (g^2(x) - g^2(k)) h(x) dx + \int_{-k}^{-a} (g^2(x) - g^2(k)) h(x) dx \right] \right]}{c \left[ 2 \int_0^k g''(x) f_0(x) dx \right]^2}$$

Noting the weak monotonicity of  $|g(\cdot)|$  implies that  $g^2(\cdot)$  is weakly increasing, we observe that the numerator of the above is minimized while the denominator is simultaneously maximized if  $h(x) = 0$  on  $[-k, -a] \cup [a, k]$ . The expression given for  $\inf_h \eta(k, h)$  in part (a) then follows directly.

For part (b) a similar evaluation when  $k \leq a$  yields

$$\eta(k, h) = \frac{c \left[ 2 \int_0^k g''(x) f_0(x) dx \right]^2}{\left[ 2 \int_0^k g^2(x) f_0(x) dx + 2(1-\varepsilon) g^2(k) \int_k^\infty f_0(x) dx + \varepsilon g^2(k) \left[ \int_k^\infty h(x) dx + \int_{-\infty}^{-k} h(x) dx \right] \right]}$$



Noting that 
$$\int_k^\infty h(x)dx + \int_{-\infty}^{-k} h(x)dx = 1 - 2 \int_0^k f_0(x)dx,$$

we obtain an expression for  $\eta(k,h)$  which is independent of  $h(\cdot)$  and which agrees with the expression given for  $\inf_h \eta(k,h)$  in part (b). QED

We remark that differentiation of the expressions of the Theorem could be employed as in [3] for the purpose of obtaining a condition on an optimal choice  $k$  as a solution of a nonlinear equation. However, we have found that for this stochastic signal situation the appropriate equations are typically so complicated that it is no more difficult to search directly over the  $\inf_h \eta(k,h)$  values. We also remark that the techniques of [3] could be employed to extend the Theorem to the case of an asymmetric nominal density; the chief cost for such an extension would be the considerably more cumbersome expressions for the  $\inf_h \eta(k,h)$  quantity.

We now consider an example which employs a zero mean unit variance sech noise nominal density, i.e.

$f_0(x) = \frac{1}{2} \operatorname{sech}(\frac{1}{2}\pi x)$  for all  $x$ .

We compare the "worst case" efficacy  $\eta_{2,w}$  of the detector of [2] (i.e. the minimal efficacy over those  $h(\cdot)$  satisfying our constraints as well as those of [2]) with the "worst case" efficacy  $\eta_{1,w}$  of our detector with  $k$  chosen via the Theorem. The results, which depend on the choice of  $a$  and  $\epsilon$ , are tabulated below:

Table: Values of  $\eta_{1,w}/\eta_{2,w}$

	$\epsilon=0.1$	$\epsilon=0.3$	$\epsilon=0.5$	$\epsilon=0.7$
$a=2.00$	6.02	2.65	1.44	1.13
$a=1.50$	6.02	2.65	1.44	1.13
$a=1.00$	3.45	2.73	1.44	1.13
$a=0.50$	2.73	1.55	1.15	1.04

Note that the most dramatic improvement in performance occurs when the greatest certainty is present concerning the univariate noise density (small  $\epsilon$  and large  $a$ ). This is probably consistent with what we might expect, however it is also directly opposite to what was observed for the Gaussian example of [3] for the time varying deterministic signal case. Other such opposing effects are known to exist when comparing the stochastic and deterministic signal cases, e.g. respective symmetry and skew symmetry of the nonlinearity  $g(\cdot)$ , and thus the results illustrated here need not be viewed as surprising when compared to those of [3].

### III. CONCLUSION

We have introduced an approach toward improving the asymptotic performance of a detector of a stochastic signal by exploiting available knowledge of the univariate noise density near the origin, while at the same time retaining the robustness of the detector to inexact knowledge of the tails of the noise density. We have shown by way of example that such improved "worst case" performance does indeed occur.

### ACKNOWLEDGEMENT

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